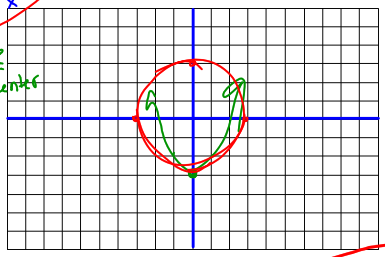
Solving Non-Linear Systems

First, make a sketch and state how many solutions you believe there to be. Then, using algebra, find the exact solutions. from the sketch we see that there should be 3 solutions

Ex. 1 $y = x^2 - 3$ parabola $y+3=x^2$

$x^2 + y^2 = 9$ circle (0,0) center $r=3$



Use subst. replace x^2 with $y+3$

$y+3+y^2=9$

$-9 \quad -9$

$y^2+y-6=0$

$(y+3)(y-2)=0$

$y+3=0$ or $y-2=0$

$y=-3$ $y=2$

$x^2=y+3$ $x^2=y+3$

$x^2=(-3)+3$ $x^2=2+3$

$x^2=0$ $x^2=5$

$x=0$ $x=\pm\sqrt{5}$

So 1 point is $(0,-3)$ so 2 points $(\sqrt{5},2)$ and $(-\sqrt{5},2)$

Solving a system is finding all points of intersection.

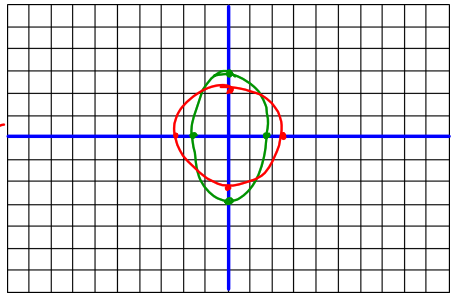
check answers in the original eqs.

4 solutions

Ex. 2 $3x^2+y^2=9$ ellipse

$x^2+y^2=5$ circle $c(0,0)$ $r=\sqrt{5}$ ≈ 2.2

$\frac{x^2}{3}+\frac{y^2}{9}=1$ ellipse



$3x^2+y^2=9$

$-x^2+-y^2=-5$

$2x^2=4$ use elimination method

$x^2=2$

$x=\pm\sqrt{2}$

Subst into $x^2+y^2=5$

$2+y^2=5$

$-2 \quad -2$

$y^2=3$

$y=\pm\sqrt{3}$

solution

$(\sqrt{2},\sqrt{3})$

$(\sqrt{2},-\sqrt{3})$

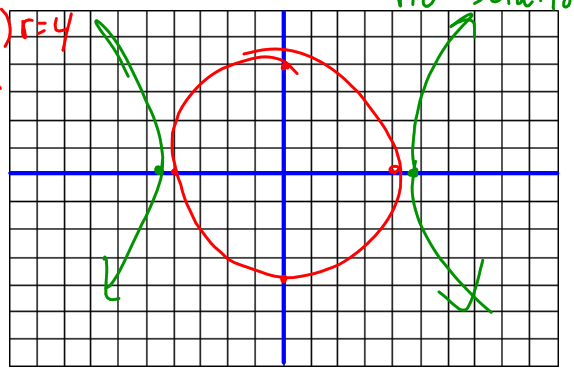
$(-\sqrt{2},\sqrt{3})$

$(-\sqrt{2},-\sqrt{3})$

Ex. 3 $x^2 + y^2 = 16$ circle $C(0,0)$ $r=4$

$x^2 - y^2 = 20$ hyperbola

$\frac{x^2}{20} - \frac{y^2}{20} = 1$



no solution

$x^2 + y^2 = 16$

$x^2 - y^2 = 20$

$2x^2 = 36$

$x^2 = 18$

$x = \pm\sqrt{18}$

subst $x^2 + y^2 = 16$

$18 + y^2 = 16$

$-18 \quad -18$

$y^2 = -2$

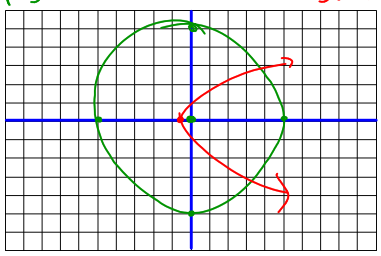
$y = \pm\sqrt{-2}$ ← no real solution.

Ex. 4 $x^2 + y^2 = 25$ circle $r=5$ $C(0,0)$

$\frac{y^2}{2} = \frac{2x+1}{2}$ parabola

$\frac{1}{2}y^2 = x + \frac{1}{2}$

$x + \frac{1}{2} = \frac{1}{2}y^2$



so 2 solutions

solving algebraically

$x^2 + y^2 = 25$

$y^2 = 2x + 1$

by subst. $x^2 + 2x + 1 = 25$

$-25 \quad -25$

$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$

$x+6=0$ or $x-4=0$

$x = -6$ $x = 4$

$y^2 = 2x + 1$ $y^2 = 2(4) + 1$

$y^2 = 2(-6) + 1$ $y^2 = 9$

$y^2 = -12 + 1$ $y = \pm 3$

$y^2 = -11$

$y = \pm\sqrt{-11}$ non-real

$(4,3)$ and $(4,-3)$